Chromatic View of 90° FODO Cell

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Chromatic effects

- The tunes vs. the energy change

$$\Delta Q_{x,y} = Q'_{x,y} \cdot \delta \tag{1}$$

- The β function vs. the energy change

$$d\beta/\delta = \lim_{\delta \to 0} \frac{\beta(\delta) - \beta(0)}{\delta} \tag{2}$$

- The D_x function vs. the energy change

$$D_x = \lim_{\delta \to 0} \frac{x_{co}(\delta) - x_{co}(0)}{\delta} \tag{3}$$

where

$$\delta = (p - p_0)/p_0 \tag{4}$$

- those quantities affect the Luminosity or Dynamic aperture.

Reduce the first order beta-beating

- There are two approaches to reduce the higher order chromatic effects:

1) B.W. Montague's approach,
reducing he defined $\mathbf{w}$ vector , CERN Yellow Reports
2) Hamiltonian approach.
reducing half-integer strength, Widemann and S.Y. Lee

- In B.W. Montague's notation, to minimize the function $\mathbf{w}$:

$$\left\{ \begin{array}{ccc} a & = & \lim_{\delta\to 0} \frac{\beta_1-\beta_0}{[\beta_1\beta_0]^{1/2}}\frac{1}{\delta} \\ b & = & \lim_{\delta\to 0} \frac{(\alpha_1\beta_0-\alpha_0\beta_1)}{[\beta_1\beta_0]^{1/2}}\frac{1}{\delta} \\ \mathbf{w} & = & \frac{1}{2}(b+ia) \end{array} \right. . \tag{5}$$

- $\mathbf{w}$ propagates at twice the phase advance.

$$\text{At quadrupole, } \Delta a = -\beta k_1 ds, \ \Delta b = 0 \ .$$

$$\text{At Sextupole, } \Delta a = -\beta k_2 D_x ds, \ \Delta b = 0 \ .$$

w vector propagation

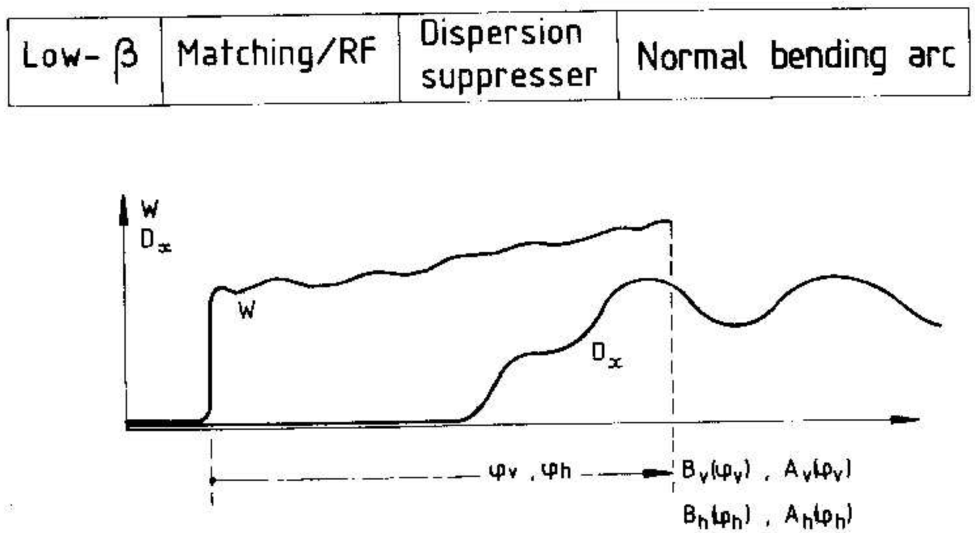


Fig. 1 Variation of chromatic perturbation W and dispersion D_x near the interaction region

Reduce the half-integer resonance strength

- In Hamiltonian Language, to reduce the half-integer resonance strengths

$$\frac{\Delta\beta}{\beta} = -\frac{\mu}{2}\sum_{-\infty}^{+\infty}\frac{J_p e^{-ip\Phi}}{\mu_0^2 - (p/2)^2} \tag{6}$$

$$Q^{''} = -Q^{'} - \frac{|J_p|^2/\delta^2}{4(\mu - p/2)} \tag{7}$$

- Higher order dispersion D_xs have similiar descriptions, see Wiedmann's book.

- J_p, $p = [2\mu]$, is the leading term to decide the high order chrmatic effects.

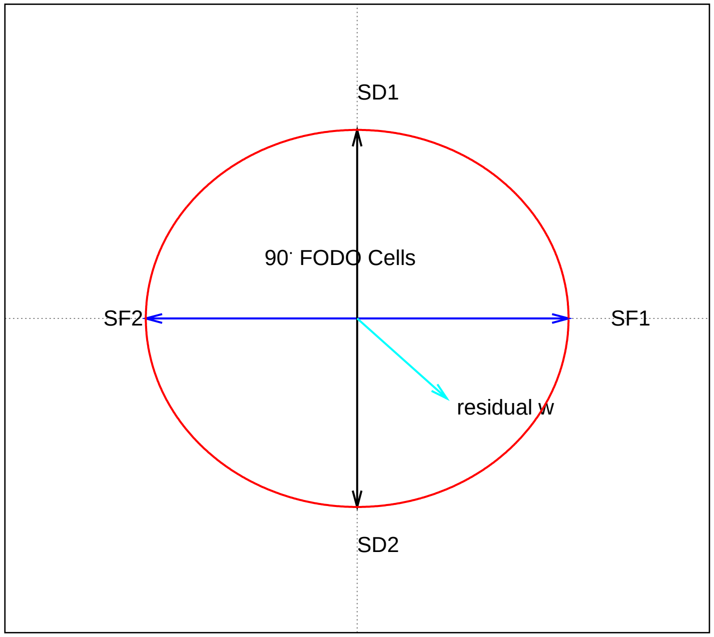
$$\left\{\begin{array}{lcl} J_{p,x} & = & \frac{1}{2\pi}\oint \beta_x \Delta K_x e^{ip\Phi_x} ds \\ J_{p,y} & = & \frac{1}{2\pi}\oint \beta_y \Delta K_y e^{ip\Phi_y} ds \end{array}\right. , \tag{8}$$

- Montague's notation and Hamiltonian approach is similar.

- Higher order Beta-beating, Dispersion, chromacticities also can be reduced.

For N repeatative FODO CELLS

- Two families (SF,SD) can correct the first order chromaticities.
- More effective families are used for the high order chromatic corrections
- For example, for 90° FODO cells, 4 sub-families are demanded.



- Effective Sorting: SF1/SD1/SF2/SD2/SF1/SD1/SF2/SD2.... ,
Each arc totally $4 \cdot p$ 90° FODO cells needed.

Further work, preliminary plan

1) 4 sextupole families scheme check (more expensive)

How much to reduce the chromatic effects,
HOw much to improve the DA?
Its benefits, worthy to add power supplies?

2) Local chromaticity correction (cheaper)

Using the sextupoles close to triplets for local correction
calculation, lattice matching,....
Make sure it works or not, then may go to operation

3) Chromaticity control

Able to continuously and non-destructively measure chromaticities
Then chromaticity modelling based on operation
Then easy to construct the chromaticity feedback